Mathematical Appendix to Special Interests, Regime Choice, and Currency Collapse

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This is the mathematical appendix that accompanies the paper "Special Interests, Regime Choice, and Currency Collapse."

Derivations

Derivation of (3) and (A.7). The derivation of price aggregators is fairly standard and only be sketched briefly here. The steps involve

$$\min_{c(z)} = \int_0^1 p(z)\, c(z)\, dz \text{ subject to } \left[\int_0^1 c(z)^{\frac{\theta-1}{\theta}}\, dz\right]^{\frac{\theta}{\theta-1}} = 1,$$

$$\min_{l(z)} = \int_0^{\frac{1}{2}} w(z)\, l(z)\, dz \text{ subject to } \left[\int_0^{\frac{1}{2}} l(z)^{\frac{\phi-1}{\phi}}\, dz\right]^{\frac{\phi}{\phi-1}} = 1,$$

for (3) and (A.7), respectively. We will consider the latter case, for we use a result in a derivation later on. The first order condition that obtains simplifies to

$$w(z)\, l(z) = \lambda \cdot \left[l(z)^{\frac{\phi-1}{\phi}}\right],$$

where λ is the Lagrangian multiplier. This expression can be further simplified to

$$l(z) = w(z)^{-\phi}\, \lambda^{\phi}.$$

Raising both sides by the exponent $\frac{\phi-1}{\phi}$, integrating over the range $\left[0, \frac{1}{2}\right]$, and simplifying gives

$$\lambda = \left[\int_0^{\frac{1}{2}} w_s(i)^{1-\phi}\, di\right]^{\frac{1}{1-\phi}}. \qquad \text{(MA.1)}$$

Solve for λ by integrating the original first order condition over the same range, and recognizing that $\int_0^{\frac{1}{2}} w_s(i)^{1-\phi}\, di = WL$, obtain

$$\lambda = W. \qquad \text{(MA.2)}$$

Substitute (MA.2) into (MA.1) to obtain the wage aggregator. □

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Derivation of (8). Given the consumption index (2), the consumer's problem is to maximize this expression subject to the nominal budget constraint

$$\int_0^1 p(z)\, c(z)\, dz = \bar{E},$$

where $\bar{E}$ is any fixed total nominal expenditure on goods. For any two goods z and z', therefore, obtain the marginal substitution condition for given by

$$\frac{c(z)}{c(z')} = \left[\frac{p(z')}{p(z)}\right]^{\theta}.$$

Substituting this expression into the budget constraint and recognizing that all expenditure is exhausted yields

$$\int_0^1 p(z')\, c(z) \left[\frac{p(z)}{p(z')}\right]^{\theta} dz = PC.$$

Using the price index (3) and simplifying then gives us the required demand function. □

Derivation of (A.9). The firm's minimization problem is to

$$\min_{w(i),\mathbf{l}} \int_0^{\frac{1}{2}} w(i)\, l^z(i)\, di,$$

subject to an output target

$$y(z) \geq \bar{Y};$$

the first order condition that results is

$$l^z(i)^{\frac{1}{\phi}}\, w(i) = \lambda y(z)^{\frac{1}{\phi}},$$

which rearranging gives

$$l^z(i) = \left[\frac{w(i)}{\lambda}\right]^{-\phi} y(z). \tag{MA.3}$$

Substituting (MA.2) into (MA.3) then gives us the labor demand function as stated. Note that it is possible to instead solve for the firm's *maximization* problem, subject to a production function given by $y(z) = \int l^z(i)\, di$. In this case labor demand will instead be represented in terms of prices and output:

$$l^z(i) = \left[\frac{w(i)}{\hat{p}(z)}\right]^{-\phi} y(z),$$

where $\hat{p} \equiv \max\{p(z), Ep^*(z)\}$. □

Derivation of (A.11). First, substitute (A.10) into (4), and then use as a constraint to maximize (1). The first-order condition simplifies to

$$\frac{1}{C^i P} \cdot (1-\phi) + \kappa\phi w\,(i)^{-1} \int_0^{\frac{1}{2}} \left[\frac{w\,(i)}{W}\right]^{-\phi} y\,(z)\,dz = 0,$$

and by substituting back (A.10) and rearranging we obtain the factor pricing equation as desired. □

Derivation of (A.12). Firms take wages as given and seek in each period to maximize

$$\pi\,(z) = p\,(z)\,y\,(z) - \int_0^{\frac{1}{2}} w^z\,(i)\,l^z\,(i)\,di,$$

subject to the production function and (10). Since households differ in their equity income, their labor supplies will, in general, differ, which means that wages received by each agent will also differ. Thus, even with the CES production technology, the production function can be highly nonlinear. In order to proceed, we assume that firms can perfectly price discriminate across countries, and so sets separate prices for Home and Foreign markets. The problem is then

$$\begin{aligned}\pi\,(z) = &\left[p\,(z) - \int_0^{\frac{1}{2}} w\,(i)\left[\frac{w\,(i)}{W}\right]^{-\phi} di\right]\left[\frac{p\,(z)}{P}\right]^{-\theta} \int_0^{\frac{1}{2}} C^i di \\ &+ \left[Ep^*\,(z) - \int_0^{\frac{1}{2}} w\,(i)\left[\frac{w\,(i)}{W}\right]^{-\phi} di\right]\left[\frac{p^*\,(z)}{P^*}\right]^{-\theta} \int_{\frac{1}{2}}^1 C^{*i} di,\end{aligned} \tag{MA.4}$$

and maximizing with respect to $p\,(z)$ and $p^*\,(z)$ then gives us the pricing rules as required.

□